Chapter

ORGANIZATION AND ESTIMATION OF STUDENTS' SELF-GUIDED WORK IN STUDIES OF MATHEMATICAL ANALYSIS

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ABSTRACT

In this article we consider algorithm of improving organization of students’ self-guided work when studying mathematical analysis. For example, we consider using L'Hopital's Rule.

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INTRODUCTION

While organization of students' self-guided work process, that is connected with a solution of exercises of mathematical analysis, it is extremely important to make the proper algorithm that allows the student to penetrate the essence of the theory consistently and consciously, that is provided to solve these exercises, analyze the conditions under which this technique can be used and cases when it is not applicable, and learn to monitor the obtained results.

In the provided study the creation of such algorithm is considered on examples of the methodological guidelines of students' self-guided work using L'Hopital's Rule for calculating the limits of functions:

1) The general principals of theory of limits are stated
2) Statement of L'Hopital's Rule
3) Applying method of L'Hopital's Rule for understanding the indeterminated forms $\left(\frac{0}{0}\right)$ u $\left(\frac{\infty}{\infty}\right)$
4) A sufficient number of analyzed and resolved exercises
5) Exercises for self-guided work
6) Answers and guidance for solving exercises
7) 7)Techniques that make it possible to convert other indeterminated forms to the form $\left(\frac{0}{0}\right)$ u $\left(\frac{\infty}{\infty}\right)$

8) Examples of solution of such exercises
9) Exercises for self-guided work
10) Answers to the proposed exercises, and guidance to the solution if it is necessary
11) Examples that draw the students' attention to the fact that L'Hopital's Rule cannot always be applied
12) Exercises for self-analysis of such situations

The study also examines the possibility of monitoring the results of students' self-guided work through Google forms. Two completely different cases can be taken whilst calculating the limits of function:

– firstly, it is a case, when there is no indeterminancy, and the limit of function, if it exists, can be calculated at once with help of limit features
– secondly, it is a case, when indeterminancy must be evaluated in order to calculate the limit,

$$\left(\frac{0}{0}\right) \left(\frac{\infty}{\infty}\right) (0\cdot\infty) \quad \infty - \infty \quad (1^{\infty}) \; (0^{0}) \; (\infty^{0})$$

Theorem (L'Hopital's Rule)

Let us assume derivatives and at a certain point, a for function f(x)и g(x)

$$g'(x) \neq 0 \; \forall x \in U(a)$$

$$\exists \; \lim_{x\to a} \frac{f'(x)}{g'(x)} = A \quad \text{where } A \leq +\infty$$

or a) $\lim_{x\to a} f(x) = 0$ and $\lim_{x\to a} g(x) = 0$

or б) $\lim_{x\to a} f(x) = \infty$ and $\lim_{x\to a} g(x) = \infty$

Then $\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)} = A$

Let us assume that limit calculation $\lim_{x\to a} \frac{f(x)}{g(x)}$ requires the evaluation of indeterminate form $\left(\frac{0}{0}\right)$ or $\left(\frac{\infty}{\infty}\right)$ and suppose that in this case all hypotheses of theorem are performed (L'Hopital's Rule).

To calculate the limit we need to find the derivative of function in numerator and derivative of function in denominator separately, and then we can cover $\lim_{x\to a} \frac{f'(x)}{g'(x)}$

If there are no indeterminate forms, we can calculate limits.

If there are indeterminate forms $\left(\frac{0}{0}\right)$ or $\left(\frac{\infty}{\infty}\right)$ the rule can be applied again (if all hypotheses of theorem are performed again).

Let us consider some examples of calculating limits using L'Hopital's Rule.

Example 1. Calculating the limit $\lim_{x\to 2} \frac{x^3 - 4x^2 + 4x}{x^3 - 12x + 16}$

$$\lim_{x\to 2} \frac{x^3 - 4x^2 + 4x}{x^3 - 12x + 16} \overset{\left(\frac{0}{0}\right)}{=} \lim_{x\to 2} \frac{3x^2 - 8x + 4}{3x^2 - 12} \overset{\left(\frac{0}{0}\right)}{=} \lim_{x\to 2} \frac{6x - 8}{6x} = \frac{4}{12} = \frac{1}{3}$$

In this example L'Hopital's Rule was applied twice to evaluate indeterminate forms.

Example 2. Calculating the limit $\lim\limits_{x\to+0}\dfrac{\ln\sin x}{\ln\sin 5x}$

$$\lim_{x\to+0}\frac{\ln\sin x}{\ln\sin 5x}\overset{\left(\frac{\infty}{\infty}\right)}{=}\lim_{x\to+0}\frac{(\ln\sin x)'}{(\ln\sin 5x)'}=\lim_{x\to+0}\frac{\frac{1}{\sin x}\cos x}{\frac{1}{\sin 5x}5\cos 5x}=\frac{1}{5}\lim_{x\to+0}\frac{\sin 5x}{\sin x}\lim_{x\to+0}\frac{\cos x}{\cos 5x}=\frac{5}{5}=1$$

L'Hopital's Rule allows to evaluate indeterminate forms $\left(\frac{0}{0}\right)$ and $\left(\frac{\infty}{\infty}\right)$ directly. To evaluate other indeterminate forms we need to modify the expression that way to get indeterminate

Forms $\left(\frac{0}{0}\right)$ or $\left(\frac{\infty}{\infty}\right)$ and after this we can apply L'Hopital's Rule.

Modifying process of some indeterminate forms to the other can be stated in rules that have symbolic representation in the following

form:1) $(0\cdot\infty)=\left(\dfrac{0}{\frac{1}{\infty}}\right)=\left(\dfrac{0}{0}\right)$ or $(0\cdot\infty)=\left(\dfrac{\infty}{\frac{1}{0}}\right)=\left(\dfrac{\infty}{\infty}\right)$

2) $((+\infty)-(+\infty))=\left(\frac{1}{0}-\frac{1}{0}\right)=\left(\frac{0-0}{0}\right)=\left(\frac{0}{0}\right)$

3) $\left(0^0\right)=\left(e^{0\cdot\ln 0}\right)=\left(e^{0\cdot\infty}\right)=(0\cdot\infty)$ and further in accordance with the rule)

4) $\left(\infty^0\right)=\left(e^{0\cdot\ln\infty}\right)=\left(e^{0\cdot\infty}\right)=(0\cdot\infty)$

5) $\left(1^\infty\right)=\left(e^{\infty\cdot\ln 1}\right)=\left(e^{\infty\cdot 0}\right)=(\infty\cdot 0)$

Example 3. Calculating the limit $\lim_{x\to+0}(x\ln x)$

$$\lim_{x\to+0}(x\ln x) \overset{(0\cdot\infty)}{=} \lim_{x\to+0}\frac{\ln x}{\frac{1}{x}}^{\left(\frac{\infty}{\infty}\right)} = \lim_{x\to+0}\frac{\frac{1}{x}}{-\frac{1}{x^2}} = \lim_{x\to+0}(-x) = 0$$

Example 4. Calculating the limit $\lim_{x\to0}\left(\frac{1}{\sin x}-\frac{1}{x}\right)$

$$\lim_{x\to0}\left(\frac{1}{\sin x}-\frac{1}{x}\right)$$

$$\overset{(\infty-\infty)}{=} \lim_{x\to0}\left(\frac{x-\sin x}{x\sin x}\right)^{\left(\frac{0}{0}\right)} = \lim_{x\to0}\frac{1-\cos x}{\sin x + x\cos x} \overset{\left(\frac{0}{0}\right)}{=} \lim_{x\to0}\frac{\sin x}{\cos x+\cos x - x\sin x} = 0$$

Example 5. Calculating the limit $\lim_{x\to+0}\arcsin x^{tg\,x}$

$$\lim_{x\to+0}\arcsin x^{tg\,x} \overset{(0^0)}{=} \lim_{x\to+0} e^{\ln\arcsin x^{tg\,x}} = e^{\lim_{x\to+0} tg\,x \ln\arcsin x} \overset{(0\infty)}{=} e^{\lim_{x\to+0}\frac{\ln\arcsin x}{\frac{1}{tg\,x}}} = \left[\frac{\infty}{\infty}\right] =$$

$$= e^{\lim_{x\to+0}\frac{(-tg^2x)\cos^2x}{\arcsin x\sqrt{1-x^2}}} = e^{-\lim_{x\to+0}\frac{\sin^2x}{\arcsin x\sqrt{1-x^2}}} = e^0 = 1$$

A known formula is used in this example $\lim_{x\to0}\frac{\sin x}{\arcsin x}=1$

Example 6. Calculating the limit $\lim_{x\to\frac{\pi}{2}-0}(tgx)^{\cos x}$

$$\lim_{x\to\frac{\pi}{2}-0}(tgx)^{\cos x}\overset{(\infty^0)}{=}e^{\lim\limits_{x\to\frac{\pi}{2}-0}(\cos x\ln tgx)}\overset{(0\infty)}{=}e^{\lim\limits_{x\to\frac{\pi}{2}-0}\frac{\ln tgx}{\frac{1}{\cos x}}}\overset{\left(\frac{\infty}{\infty}\right)}{=}e^{\lim\limits_{x\to\frac{\pi}{2}-0}\frac{\frac{1}{tgx}\frac{1}{\cos^2 x}}{-\frac{1}{\cos^2 x}(-\sin x)}}=e^{\lim\limits_{x\to\frac{\pi}{2}-0}\frac{\cos x}{\sin^2 x}}=e^0=1$$

Example 7. Calculating the limit $\lim\limits_{x\to 1} x^{\frac{1}{x-1}}$

$$\lim_{x\to 1} x^{\frac{1}{x-1}}\overset{(1^\infty)}{=}e^{\lim\limits_{x\to 1}(\frac{1}{x-1}\ln x)}\overset{(\infty 0)}{=}e^{\lim\limits_{x\to 1}\frac{\ln x}{x-1}}\overset{\frac{0}{0}}{=}e^{\lim\limits_{x\to 1}\frac{\frac{1}{x}}{1}}=e^{\lim\limits_{x\to 1}\frac{1}{x}}=e$$

It is important to remember, that L'Hopital's Rule can be used only in that case, when there is a ratio limit of derivatives of functions i. e. $\exists\lim\limits_{x\to a}\frac{f'(x)}{g'(x)}=A$ (finite or infinite).

If there is no such ratio, the other means should be used to calculate the ratio limit of the functions or prove its nonexistence.

Example 8. Calculating the limit $\lim\limits_{x\to\infty}\frac{x+\sin x}{x}$

In this case we have an indeterminate form $\left(\frac{\infty}{\infty}\right)$ and we cannot use L'Hopital's Rule such as $\lim\limits_{x\to\infty}\frac{(x+\sin x)'}{(x)'}=\lim\limits_{x\to\infty}\frac{1+\cos x}{1}=\lim\limits_{x\to\infty}(1+\cos x)$ does not exist at all.

With a direct calculation of the limit, we obtain

$$\lim_{x\to\infty}\frac{x+\sin x}{x}=\lim_{x\to\infty}\frac{x(1+\frac{\sin x}{x})}{x}=\lim_{x\to\infty}(1+\frac{\sin x}{x})=1,$$

such as $\lim_{x \to \infty} \frac{\sin x}{x} = 0$

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