On the Up-to-date Course of Mathematical Logic for the Future Math Teachers

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Abstract All-round development of the everyday logic of students should be considered as one of the most important tasks of general secondary education on the whole and general secondary mathematics education in particular. The article is devoted to the problem of organization in teachers' training institutions of higher education the expedient training for the future mathematics' teachers in institutions of general secondary education to ensure their ability to realize during all their future professional activities the necessary participation in forming the everyday logic of their pupils. The authors think that vocational educational program of training the future secondary school teachers of mathematics must contain a separate course of mathematical logic including at least 90 training hours (3 credits ECTS). Although the content filling of the course cannot be irrespective of the general level of arrangement of mathematics education in the corresponding country, it ought to be a subject of discussion of the international mathematics community and managers in the sphere of higher mathematics education. Simultaneously, the role, the place, and the expedient structure of such a course in the corresponding training programs should be under discussion. The article represents the authors' point of view on the problems, indicated above. The research has a qualitative character as a whole. Only some of its conclusions have statistical corroboration.

Keywords Mathematical logic, everyday logic, program of training the future teachers of mathematics

1. Introduction

It seems to make no sense to discuss what should be recognized as the primary in the history of humanity – the mathematical knowledge or those deductions of generalizing character with the help of which this knowledge has been received. It is quite clear that practical experience precedes any knowledge as well as any kind of generalizations. Mathematical knowledge, as well as all other knowledge of special character, has not been picked out from the general amount of human knowledge at the stage of humanity's beginnings. At the same time, the existence of one or another conclusion is, as a matter of a fact, a unique illustration of the existence of such deductions that have made these conclusions possible. Conclusions of logic, as a science about laws of human thinking, have been obtained during the process of forming human civilization as a result and, at the same time, as a necessary precondition of its further development. The ability of logical thinking and human knowledge about the properties of the human's environment have formed simultaneously anyway by the fact that a man itself represents a part of this environment. We know logic, as a branch of philosophy. We know formal logic, as such part of logic that establishes laws of creating new knowledge on the basis of the previously received excluding reference in each specific case to practical experience, mathematical logic, firstly in the form of the only one, and then – in the form of a lot of axiomatic theories. All of them have been formed as a quintessence, as a result of the development of the so-called everyday logic.

It is clear that axiomatic theories of mathematical logic on their own have appeared like formal axiomatic theories.

Though in mathematics on the whole the first examples of axiomatic theories, that create the ground of the consequent modern concepts of axiomatics and axiomatic theory, have not been formal.

As it is well known ([1], for example) axiomatic theory is called formal, if its axiomatics contains as its component some axiomatics of mathematical logic, just the axiomatic content of the theory of which exclusively is used for building propositions of the considered axiomatic theory. Otherwise, if axiomatics of the corresponding axiomatic theory doesn't contain any axiomatics of mathematical logic as its sub axiomatics, conclusions of the theory are formed exclusively with the help of the everyday logic, the theory is called informal.

The overwhelming majority of axiomatic theories of classic mathematics are informal. For the whole process of development of mathematics as a science, such fact must be recognized as quite natural. By no means such fact doesn't prejudice as their high quality constructing, as the all-round development of their possible practical applications. Creation and further development of formal axiomatic theories are necessitated, firstly, by the speeded-up development of the up-to-date information technologies. It can't be by another way in this process because, in contrast to a man, a machine can realize only the uniquely determined succession of steps of concrete operations. But for a man such detailed step-by-step realization of the necessary conclusions often is important from the point of view of the possibility of their realization and, simultaneously, often is not expedient from the point of view of possibility of realizing the essence of such conclusions. A man is able and has an attraction for "block" thinking. Maybe, heuristic thinking represents the limit form of its realization. The excessively detailed elaboration often puts obstacles to the development of the corresponding theory, though it is necessary for the whole conviction in the correctness of the obtained conclusions.

It is well known that the steps of forming the child's everyday logic in a brief, approximate, essentially accelerated variant repeats the historical steps of forming the everyday logic of all humanity. A child is forming onto a grown man only by the condition of the human surroundings. Education, just mathematics education, plays in this process an important part.

If we speak about institutions of general secondary education, the next methodical problems ought to be considered as actual.

1. To determine methods with the help of which the math teacher can promote development the everyday logical thinking of students during the teaching-learning process.
2. To answer a methodical and simultaneously mathematical question about being expedient on the whole and, in the case of the positive answer, at what stage of the teaching-learning process, in what amount, it is expedient to include elements of mathematical logic to the content of mathematical courses of institutions of general secondary education.
3. To answer the question of expediency to pick out mathematical logic as a separate educational subject for those competitors of higher education that are going to receive the qualification of a math teacher in institution of general secondary education, and, if it is so, to determine the corresponding year of training and the corresponding expedient content of the course.

The solution of the first problem seems to be almost evident for the overwhelming majority of mathematicians that are researchers and is not evident at all for the considerable amount of mathematicians that perform themselves as methodologists and managers of education. The point of view of the latter, that content of the course or the courses of mathematics at institutions of secondary education must consist exclusively of the finite quantity of facts from arithmetic, algebra, Euclidean geometry, maybe calculus, theory of probability, and mathematical statistics, is false. In spite of the situation that the amount of such facts that are accepted in different countries is different, their general quantity is more than enough (from the infinity that exists). We will not insist that an ordinary man in the absence of daily training simply is not able to remember them but will emphasize that there is no necessity in it at all.

From the very beginning of the time when a child begins speaking the question "why?" becomes his natural, favorite question. In contrast to physics, chemistry, biology and geography, mathematics directly cannot be called a science of nature. Thus, explanations of the type that "it happens so in the environment" are not regarded here. The practical-orientated training is one of the modern educational trends. In contrast to the sciences, mentioned above, mathematics has not only external but has also and, maybe, primary, internal sources of development. Thus, for mastering the corresponding material there is no necessity to remember everything. It is enough to learn only the basic statements, all the other facts can be received on their ground exclusively only with the help of logical conclusions. If we mean mathematical courses of institutions of general secondary education, the everyday logic for them will be quite enough. Application of this logic in order to receive the corresponding deductions also represents by itself a process of practical training, a process of practical training in development exactly the everyday logic, thus, the process of practical-orientated training. Proofs and substantiations form the essence of mathematics as a science. Thus, mastering at institutions of general secondary education mathematics as an educational subject without purposeful forming of the corresponding students' everyday logic is simply impossible. Just such forming constitutes the ground of the concept of so popular and so necessary for today inquiry-based learning.

Some methodologists of mathematics and managers of education support the point of view that the list of educational subjects for institutions of general secondary education must contain the discipline called "L ogic". They don't mean by this a mathematical logic, it seems, they mean some elements of the everyday logic, undoubtedly not logic as a part of philosophy. The authors consider such an approach to the process of forming the everyday logic of students of institutions of general secondary education as wrong from the methodical point of view. The everyday logic should be being formed during the process of mastering every educational discipline, and mathematics plays there the first fiddle.

The article is devoted to the problem of organization at teachers' training institutions of higher education the expedient training for the future mathematics' teachers at institutions of general secondary education to ensure their ability to realize during all their future professional activities the necessary participation in forming the everyday logic of the corresponding pupils.

We must emphasize for all this that if we mean state institutions of general secondary education or, even, private ones, school-leavers of which have a claim on a certificate of general secondary education of the state standard, not everything depends on the teacher. The fundament of a system of such training is formed by determined by the state programs educational content on mathematics and the amount of class-study hours that are planned for its mastering.

In general, our research has a qualitative character. Only some of its conclusions have been corroborated by statistics.

2. Materials and Methods

We will consider a system of training by the first (bachelor) level at institutions of higher education of the future math teachers for institutions of general secondary education. Let us suppose that the corresponding training takes four academic years, certificate of bachelor's degree received as a result of mastering the corresponding educational program, admits providing teaching-learning process on mathematics at the level of the base general secondary school. It is a question of principle if it makes sense to include for the corresponding students the course of mathematical logic to the list of required courses. Our experience tells us that in this case, in contrast to the educational programs for institutions of general secondary education, the answer must be "yes".

It must be accepted that just mathematical logic forms theoretical grounds of conclusions of mathematical courses of institutions of general secondary education that are in reality represented at these courses as conclusions received on the basis of the everyday logic. It is clear that it can't be meant mathematical logic as a science on the whole. Firstly, we can and must mean the basic, the most fundamental statements of mathematical logic as a science, and, the latter, such its statements that in the first place find their applications in mathematical courses of institutions of general secondary education. The expedient up-to-date content of such a course represents one of the main questions that should be discussed by the mathematical and methodical community.

As a result of the corresponding analysis, our institution of higher education for such a course, intended for the future teachers of mathematics at institutions of general secondary education, suggests the following content, expected to be taught in 90 educational hours that are organized into 3 content modules.

Content module 1. The origin of mathematical logic. Algebra of propositions

Theme 1. Forming of mathematical logic as a science. Logic of propositions. Interpretations of propositions

Aristotle's logic. Meaningful load of logical terms. Forming of mathematical logic in the process of solving contradictions in the theory of sets. Mathematical logic as the apparatus for creating the foundations of mathematics and formalization of mathematical and other scientific theories, as the theoretical base for creating different programming languages in computer science. Formal language and its components. Alphabet of logic of expressions. Expressions and propositions in logic of statements. Mathematical methods of conducting proofs by induction. Logical length of a proposition, proof by induction for propositions. Elementary and composite propositions. Sub formulas. Conditions of omitting parentheses in propositions. Two-element Boolean algebras. Definitions and truth definitions, their unique existence. Types of propositions: tautologies, executable and contradictory propositions. Table of basic tautologies. Equivalence of propositions. Constructing of truth tables. Determining the type of a proposition, equivalence of propositions with the help of truth tables. Non-contradictory and contradictory sets. Logical consequences of a set of propositions. Interpretations of a set of propositions. Properties of logical consequences form a set of propositions and a set of its interpretations. Determination of logical consequences of a set of **propositions** with the help of truth tables.

Theme 2. Completeness of sets of logical connections

Completeness of the set, that consists of disjunction, conjunction and negation. Literals, disjunctive and conjunctive expressions. Disjunctive normal form (DNF). Conjunctive normal form (CNF). Perfect DNF and KNF. Completeness of the set that consists of disjunction and negation. Completeness of the set that consists of conjunction and negation. Complete one-element connection sets: Pierce's arrow and Schaefer's stroke.

Content module 2. Methods of proofs

Theme 3. Method of proofs by Beth. Axiomatic method of proofs

Atomic semantic tables. Constructions of semantic tables. Contradiction of a branch and a semantic table. The property of a semantic table to be closed. Determining the truth of the proposition with the help of semantic tables. The main requirements and characteristics of systems of axioms: independence, completeness, consistency, solvability. Axiomatics of logic of propositions. The rule of inference in the logic of propositions. Derivation from a system of axioms and a set of propositions. Replacement of equivalent sub formulas. The theorem of deduction.

Theme 4. Method of resolutions

Disjunctive propositions and disjuncts. An empty disjunct. Construction of KNF with the help of truth tables. Construction of KNF with the help of replacing equivalent sub formulas. The form of writing down a disjunct. Horn's disjunct. A set of disjuncts. Logical false of an empty disjunct. Logical truth of an empty set of disjuncts. Introduction of the resolvent of two disjuncts. Substantiation the method of resolutions. The truth of the resolvent of two disjuncts. Resolvent of a set of propositions. Resolutive deduction. Examples of deductions by the method of resolutions. Determining the contradictoriness of sets by the method of resolutions.

Theme 5. Characteristics of the method of deductions

Completeness of an arbitrary method of deductions. Correctness of an arbitrary method of deductions. Completeness and correctness of axiomatic deductions from the set of propositions. Compactness of the method axiomatic deductions. Completeness and correctness of the method of resolutions. Consistency of truth valuation with the branch of the semantic table. Hintiki's lemma. Construction of truth valuation that is consistent with not contradictory branch of the semantic table. Method of proof by induction for semantic tables. Completeness and correctness of proofs under Beth. Deductions from a set of propositions with the help of semantic tables. Completeness and correctness of the method of deductions from hypotheses. The followers and the direct followers. Proper vertices of semantic tables. Finite degree of branching of semantic table. Koening's lemma. Theorem on the compactness of the method of deductions from hypotheses.

Content module 3. Logic of predicates

Theme 6. The language of predicates logic

Content and description of the basic concepts: predicates, quantifiers, constants and variables. N-digit predicates and functions. Formal language of predicate logic. Alphabet as the logical and special groups of symbols in logic of predicates. Alphabet of arithmetic of natural numbers. The law of duality for quantifiers.

Theme 7. Formulas in logic of predicates.

Substitution sets. Terms and basic terms. Elementary formulas and formulas. Sub terms and sub formulas. Bound and free inclusions of a variable to a formula. Bound and free variables in a formula. Sentences. Introduction of substitution sets, compositions of substitutions. Properties of substitutions. Obtaining new sets of formulas with the help of substitution sets. Variants. Renaming of variables.

Theme 8. Axiomatic method of deducting in logic of predicates. Disjuncts in logic of predicates. Semantics in logic of predicates

Axiomatics of logic of predicates. Rules of deducting in logic of predicates: modus ponens and the rule of generalization. Deducting formulas from the given system of axioms and from the set of formulas, equivalent replacement of sub formulas. The theorem of deduction in logic of predicates. Literals and disjuncts, various forms of writing disjuncts, Horn's disjunct in logic of predicates. Interpretations of predicate logic language. Area of interpretation. Interpretations of predicates, functions and constants. The truth of terms and sentences in the given interpretation. Elementary extensions of formal language of logic of predicates. Elementary extensions of interpretations. Models. Theories. Types of sentences. Executable sentences. Executable sets of sentences. Contradictory sets of sentences. Formulas, meaningful in general. Logical corollaries of a set of formulas. Constructing the Prenex normal form, equivalent to the given sentence. Constructing the Skolem's normal form, equivalent to the given sentence. Erban's interpretations. Erban's universe. Criterion of executability of an arbitrary interpretation. Theoretical-set form of writing the Skolem's normal form.

It is clear that for the necessary self-guided work the corresponding sources of information (see, for example, [2-10]) are indicated.

The second problem under discussion is the question of when during the four years of training we must suggest the indicated course to the corresponding students. We think that it will be better to choose the second term of the second year of their training. It is supposed that during the first year and the first term of the second year of training the corresponding students will master classical courses of higher mathematics with the help of the everyday logic, formed along the years of their previous training, will continue its thorough development during mastering these courses.

The next problem is in developing the expedient technologies of implementation of such a course, technologies of assessment the results of its mastering. The authors consider that at the beginning of mastering the course the students must pass through the diagnostic assessment on the level of formation of their ability to the everyday logic thinking. We have in mind the system of training of the future mathematics teachers, thus, the content of the corresponding assessment must have mathematical character. It may concern mathematical courses of institutions of general secondary education together with the mathematical courses of higher mathematics that the students have mastered previously. It is also clear that the corresponding assessment is impossible to carry out in the form of tests. So it must be a little class control work, time, allotted to its fulfilment, must be limited. There is a variant of such a control work. It consists of five tasks of the next type.

1. To formulate the contradiction to the given statement. (It is assumed that as a given statement we choose a statement that "on the language of logic" is written down with the help of both quantifiers (the existential quantifier and the universal quantifier) that are used several times. For example, the next statement can be suggested: "real number a is a limit of a sequence of numbers (a_n) if for every positive real numbers ε there exist such natural number n_0, that for all natural numbers that are more than n_0 the next inequality takes place: $|a_n - a| < \varepsilon$.

2. To formulate the inverse to the given statement. (For example, statements, that represent some theorems of Euclidean geometry, can be proposed here. It is not important by this, the given statement is true or false, the statement, inverse to the given one, is true or false. We can offer the next one as a concrete example: "if a circle can be inscribed into some quadrangle then the sums of the opposite sides of this quadrangle equal each other".) Suppose that in the process of mastering the course with the help of the modern innovative technologies we can realize the momentary examination of the given answers to the tasks of the mentioned types. Then, in the case of the wrong answer, there is a possibility to suggest the corresponding student "to compose" the right answer from the presented word-combinations, in the case of repeating the wrong answer – to suggest the student to choose the right answer from the given five possible ones.

3. To prove the truth of the given numerical inequality for some corresponding amount of arbitrary real numbers. (There can be suggested, for example, such inequalities as

$$\frac{a^2 + b^2}{2} \geq ab \quad \text{or} \quad |a - b| \leq |a - c| + |c - b|.$$

The classical mistake in the process of proving the truth of such inequalities is the situation when the student conducts reasoning only "in one direction", from the given inequality to the inequality, the correctness of which is evident. Such proof is wrong. The right proof requires carrying out the reasoning in the opposite direction. At the same time, we can't often understand preliminarily from exactly what right inequality it is better to start. By the fact, usually, we have "to move" in both directions, from the inequality that must be proved to the evidently right inequality and only then – in the opposite direction. The way of providing the proof "from the contradiction" seems more efficient.)

4. To formulate the theorem, inverse to the given one, to prove it by the method of "logical inevitability". (For example, the next theorems of Euclidean geometry do well for the task. 1. Sides of the triangle, opposite to its equal angles, are equal to each other; a side of the triangle, opposite to the bigger of its two angles, is bigger than the side, opposite to the smaller of them. 2. Two incline segments are drawn to the straight line from a point that doesn't belong to it. If the incline

segments are equal to each other, then their projections onto the line are equal too; projection of the bigger incline segment is bigger.)

5. A hundred of the first-year students are mastering the training material at some college. 65 of them can play football, 70 of them can play volleyball and 75 of them can play basketball. Indicate the lowest possible number of students that can play all three games.

The task of the last type, in fact, is a task of the set theory. Theoretically, it is supposed that the students have mastered elements of the set theory during their first year of training. Thus, we can expect that they are able to find the necessary solution. It is interesting what explanations will be produced for it.

Nowadays managers of education have some concepts about the percentage of training hours allotted for self-guided work over the content of that either the other educational discipline, according to its status, the corresponding level of education, the corresponding year of training. Somebody insists it is necessary to be no less than half of the general amount of allotted training hours, somebody thinks that it must be no less than two-thirds. The authors' point of view is the next. Taking into account the individual-orientated process of training it is impossible to determine the expedient number of such hours previously, even when the corresponding methodical support is well known. The fact has to be determined as a corollary of the described above diagnostic assessment. The number of training hours, allotted for the self-guided mastering some material of the course, must be inversely proportional to its results.

During a period of mastering any of three content modules of the course, the students are suggested, as self-control, to formulate answers to some questions onto the corresponding training material. At the same time, the students know that at the end of mastering the training content of the corresponding module they will be suggested to answer questions in the test form on the theoretical and practical components of the module. The previous self-control questions form grounds for such subsequent questions in the form of tests. A system of 30 questions for the self-guided work has been developed to the content every of three modules. As an example, the first five of them on the content of the first module are the next.

1. What is the object of investigation in Aristotle's logic?
2. What is understood by a proposition in Aristotle's logic?
3. Name all the logic connections. Cite their definitions. What symbols are used usually for their designations?
4. What types of propositions are there exist in Aristotle's logic?
5. What proposition is called an elementary? What number of sub formulas can have an elementary proposition?

Two variants of tasks in the form of tests have been developed for any of the three modules. Each variant of them contains 30 test questions. We will give some examples of the questions in the content of module 1.

1. What expression of the given ones is a proposition?

a) Every real number satisfies the inequality $x^2 \geq 0$.
b) Long live mathematics!
c) Please, open the training book on page 11.
d) Solve the equation $x^2 + 7x + 1 = 0$.

2. What we mean by a proposition in Aristotle's logic?
   a) A narrative sentence
   b) An interrogative sentence
   c) An imperative sentence
   d) A rhetorical sentence
3. How many logical connections there exist in propositional logic?
   a) Three
   b) Four
   c) Six
   d) Five
4. By what symbol do we usually denote disjunction?
   a) $\rightarrow$
   b) $\vee$
   c) $\leftrightarrow$
   d) $\wedge$

5. When is a disjunction of two propositions false?

a) When at least one of the two propositions is false

b) When both of these propositions are false

c) When one of the propositions is false, and the other is true

d) When the first proposition is true and the second one is false

6. We consider propositions α: an object X has some property P; ß: an object X has some property Q. How is formulated the proposition α→ ß?

a) If the object X has the property P, then the object X has the property Q

b) The object X has at least the property P or the property Q

c) The object X has the properties P and Q

d) The object X has not the property P

Carrying out the described above control work in the form of tests is planned during the last hour of the last of the corresponding content module practical study. The right answer to any test task is assessed by 1 number. The student receives 0 numbers in the case of a wrong answer.

We suggest the students to carry out summative control work at the end of the course. Its structure resembles the structure of the control work of the diagnostic assessment. Carrying out such work is planned during the hours of the last lecture class. The work contains five tasks, fulfillment of the every one of them is assessed from 0 to 2 numbers. Twenty variants have been worked out for such a control work.

3. Results and discussion

According to the above-mentioned content, according to the above-mentioned form, the course of mathematical logic have been mastered by a group of 18 students of the first (bachelor) level of higher education of the second year of training by the specialty “Secondary education (Mathematics)” during 2019-2020 academic year. During 2018-2019 academic year, analogously, 16 students of the second year of training have mastered the course of mathematical logic by another scheme. Only the content of the summative control work for both groups of students has been the same. In the percentage, the amount of numbers, received by students of both groups is represented in Table 1.

Table 1. Group statistics of the summative control work of experimental and control group

Numbers	Academic year	
	2018-2019	2019-2020
0	6%	-
1	6%	-
2	12%	-
3	-	-
4	13%	-
5	19%	6%
6	19%	11%
7	13%	17%
8	6%	33%
9	6%	22%
10	-	11%

The results of the investigation admit to hope that the chosen scheme of mastering at the corresponding institutions of higher education the course of mathematical logic by the future teachers of mathematics at institutions of general secondary education is successful. It is clear that it can’t ensure on the whole the corresponding students to become proficient in all methodical aspects of development the everyday logic of students at institutions of general secondary education during mastering mathematics. The corresponding tasks also ought to be entrusted onto the course of methods of teaching mathematics at institutions of general secondary education.

4. Conclusions

The all-round development of the everyday logic of students is one of the most important tasks of general secondary education on the whole and general secondary mathematics education in particular. It is also one of the most important tasks of the profile education in colleges (see [11], for example). The vocational education program of training in teacher's training institutions of higher education the future teachers of mathematics for institutions of general secondary education, as also for colleges, must contain a separate course of mathematical logic expected to be taught in 90 training hours (3 credits ECTS). Undoubtedly, the content filling of the course depends on the general level of arrangement of mathematics education in the corresponding country. At the same time, it must be a subject of discussion of the international mathematics community and managers in the sphere of higher mathematics education. Simultaneously the role, the place and the expedient structure of such a course in the corresponding training programs are a sense to be under discussion.

The article represents the authors' point of view on the above-mentioned problems. Also, some elements of its approbation are described that in the authors' opinion confirm the expediency of its realization.

REFERENCES

[1] Fraenkel A, Bar-Hillel Y., "Mathematical and semantic approaches" in Foundations of Set Theory, Mir, 1958, pp. 317-416.

[2] Halmos P., "Naive Set Theory", Paperback edition , 2011, pp. 1-112.

[3] Kleene's S. C., "Introduction to Metamathematics. Seattle", Ishi Press, 2009, pp. 1-550.

[4] Kolmogorov A. N., Dragalin A. G., "Introduction to mathematical logic", Moskovskii Gosudarstvennyi Universitet, 1982, pp.10-116.

[5] Metakides G., Nerode A, "Propositional logic" in Studies in Principles of Logic and Logic Programming Computer Science and Artificial Intelligence, vol. 13, Elsevier Science, 1996, pp. 1-112.

[6] Novikov P. S., "Elements of Mathematical Logic", Oliver and Boyd, 1964, pp.1-296.

[7] Pascal H., Jens L., P. Axel, "Logics for the semantic web" in Handbook of the History of Logic, Elsevier Science, BV., vol.8,pp. 679-610, 2014. DOI: https://doi.org/10.1016/B978-0-444-51624-4.50016-2

[8] Smith, R, "Aristotle's Logic", The Stanford Encyclopedia of Philosophy (Fall 2020 Edition), Edward N. Zalta (ed.), URL :https://plato.stanford.edu/archives/fall2020/entries/aristotle-logic/.

[9] Stoll R. R., Freeman W. H. and Co, "Logic" in Sets, Logic and Axiomatic Theories, 1st Edition, W. H. Freemanand Co., 1961, pp.72-206.

[10] Turing, A. M., Gandy, R. O., C. E. M Yates., "On computable numbers, with an application to the Entscheidungs problem". In Mathematical logic: Vol. Collected works of A. M. Turing, 1st ed, Elsevier Science, 2001, pp. 9–56. DOI: https://doi.org/10.1016/B978-044450423-4/50005-X

[11] Villamar A. J., Gayagoy M, Matalang F., K. J. Catacutan, "Usefulness of Mathematics Subjects in the Accounting Courses in Baccalaureate Education", Mathematics and Statistics vol. 8, no. 1, pp. 27-31, , 2020. DOI: 10.13189/ms.2020.080103