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DEVELOPMENT OF ABSTRACT THINKING OF FUTURE MATHEMATICS SCHOOL-TEACHERS

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Abstract. *The author presents such content of course of linear algebra for future Mathematics school-teachers that, by his experience, can ensure the necessary development of abstract thinking of the students. The author offers to pay attention to the concepts of axiom, theorem and lemma, to select certain types of theorems and certain methods of its proof at the beginning of the course. Further, it makes sense to consider the theory of matrices and systems of linear algebraic equations. A special attention is paid to the technique of introducing a concept of a group. The proposed system has been tested. The approbation confirms the fact that such a system promotes the necessary development of students' abstract thinking.*

Keywords: *linear algebra, mathematical theory, a group, a set, mathematical logic, an example.*

Introduction. Linear algebra is one of subjects that are included to the list of mathematical disciplines for the beginning of higher education of future Mathematics and Physics school-teachers. A lot of the first-year students turn out not ready for perception and mastering large amounts of information during a lecture. Besides this it appears difficult for them to understand the essence of mathematical concepts with a high level of abstraction. Firstly it concerns to the concepts of equivalence relation, an arbitrary algebraic operation, algebraic structure and so on.

Brief review of previous publications by the theme. There are a lot of scientific and educational publications devoted to the theoretical material of different parts of linear algebra [1-6]. Some translated editions, some works of Ukrainian and Russian scientists are among them. Every edition, in fact, suggests some variant of construction a corresponding part of an educational course. In fact, Ukrainian teachers of higher school mainly use Russian editions. This is caused by economic problems as well as by large amounts of translation work. The task of creating a modern Ukrainian-language course of linear algebra remains relevant. Discussion of content of the course is still necessary.

The purpose of the article is to present and to substantiate some consistence of construction the most appropriate for future Mathematics school-teachers present-day course of linear algebra.

Materials and methods. Our experience tells us that we must start such course of linear algebra with the theory of matrix, in particular, with operations on them, with common information about a system of linear algebraic equations and the method of Gauss of their solving. It allows us to use only such basic concepts as a number or an equation. These concepts are known to the first year students from secondary school. It is evident, that during the first lectures we must devote our attention to the concepts that are not directly related to the mathematical course we are teaching. Differences between definitions, axioms and theorems must be explained first of all. We must point out some types of theorems. Criterion of existence and uniqueness, lemma, corollary and others are among them. We must tell the students about some methods of proof. We mean a proof by contradiction, a constructive method of grounding and, later, the method of mathematical induction. Students must not form ideologically false point of view about Mathematics as a collection of several infinite chains of definitions and theorems. They must not think that anyone can begin his mastering Mathematics from any theme depending on his school training. We must take care about all of this. Students should understand that any mathematical discipline consists of many theories. Thus, during mastering the course of algebra, students are introduced to the theory of matrices and determinants, the theories of permutations, groups, rings, fields, complex numbers, vector spaces, polynomials, and so on. As an educational subject, linear algebra also includes elements of set theory together with theories of relations and mappings, elements of the theory of numbers, number systems of calculus and so on.

We must explain to the firs-year students, at least on an ordinary level, that any scientific theory starts with a set of initial concepts. Such concepts haven't definitions. They are the most basic. They can't be defined by any other simpler concepts. We can only describe them verbally, by using synonyms. A system of axioms, that binds together these initial concepts, is introduced then. Axioms are considered being true always. They do not require any proof. Some demands exist to the system of axioms. We may speak about its inconsistency, closure, completeness and correctness. Some theories include rules of receiving new statements. Such theories are called formal or semantic. For example, all theories of mathematical logic are semantic. Other mathematical theories use everyday rules of receiving new statements. They are called informal. Thus, the grounds of any mathematical theory are based on conceptions of the theory of sets and mathematical logic.

First- year students master all mathematical theories in form of naive theories or in form of informal theories.

Group theory is taught thus in such a way. This theory plays one of the most important roles in formation high level of abstract thinking and mathematical world outlook. This theory is the best one in expressing the basic idea of modern algebra. The idea supposes investigation of sets together with some system of algebraic operations and different relationships on them. There are some notes about teaching theory of groups.

The concept of a group is one of the most fundamental in Mathematics. Theory of groups represents one of the main parts of modern algebra and modern mathematics in a whole.

The concept of a group is introduced in course of linear algebra to the students that will be Mathematics school-teachers in future. Later the received knowledge is fixed by the course of algebra and theory of numbers, by other mathematical courses. Groups of different nature and different properties are found in almost all parts of mathematical course of secondary school. We use groups of numbers, groups of transformations of a plane and of a space that include groups of symmetries of geometric figures, groups of functions and so on there.

For future Mathematics school-teachers it seems expedient to generalize and to deepen their knowledge of the theory of groups within the corresponding elective course. Three sources form the historical foundations of group theory. They are a problem of solving in radicals algebraic equations of the order $n \geq 5$; works of L. Euler, devoted to remainders, and works of K. Gauss, devoted to composition of binary quadratic forms, relationships between different geometrical theories by the keep of groups of transformations.

The golden age of group theory starts from the end of the nineteenth century. At this time the main results of the theory have been obtained, the main directions of its future development have been created. At the same time, mathematicians have pronounced a lot of previous restrictions, for examples, such as finite character of a group or finite character of orders of elements of a group. Thus, the concept of a group has gained a modern abstract content. From this time by a group we understand a set of arbitrary nature with some binary algebraic operation that satisfies well-known axioms. A concept of a group can be introduced by several means, with the help of equivalent definitions. Thus, students receive an opportunity to understand that different approaches in introduction of the same concept exist. It also allows us to consider different criteria of groups and to prove some basic properties of groups. The uniqueness of neutral element, the uniqueness of inverse element for any element of a group, general associative law, properties of inverse element and the law of reductions are among them.

Therefore, before the introduction of the concept of a group, we must give a definition of a binary algebraic operation and cite enough quantity of examples of maps that are such operations and that aren’t. So, before the study the theory of groups, students need to master the concept of a mapping, different types of mappings, composition of mapping and invers mapping also. In addition, they need to know some elements of the theory of matrix and some elements of the theory of substitutions. This is explained by the fact, that examples of groups should be different, not only of an algebraic character, but also of a geometric character and of the course of mathematical analysis. The theory of substitutions is an important independent branch of algebra. It gives examples of groups and at the same time represents a method of investigation of groups. It is proved, that any finite group is isomorphic to some group of substitutions. We must mention that without the theory of substitutions it is impossible to introduce the concept of determinant, to investigate the basic properties of determinants and to ground some methods of calculation of determinants.

During teaching we must demonstrate similarity and difference in algebraic properties of groups. Commutative and non-commutative groups, finite and infinite groups must be among the considered examples. Students must not form a mistaken notion, that any set with a binary algebraic operation is a group. Thus examples of sets with a binary algebraic operation, that aren't groups, should be given. It may be semi-groups or quasi-groups, for example. If we want to introduce such a set, we must supply the corresponding algebraic operation by some additional conditions that are called axioms. Semi-groups, quasi-groups, loops and groups are introduced with the help of axioms. A table of Kelly gives a way of introducing groups. With the help of such a table we may introduce an algebraic operation on a finite set with a small number of items.

It should be noted, that the change of algebraic operation or a set, on which it is given, necessarily leads to a change of the corresponding algebraic structure. The structure of a group may be lost. When we introduce powers and multiples of an element of a group, we should emphasize coincidence of properties of the last ones with the corresponding properties of powers and multiples of real numbers. The justice of these properties follows from the associative law, not from the nature of elements of the group.

During consideration the concept of a subgroup we should speak about tests that are convenient for determining subgroups of a specific group in practice and about the test that is convenient for theoretical proofs. When we consider some examples of groups, we must mention, which of them are subgroups of other groups. It is necessary to pay attention to the unit subgroup and the group by itself that are subgroups of every group. Alternating subgroup of substitutions and multiplicative group of matrices, whose determinant equals to unity, give important examples of subgroups. Then there is a sense to consider the concept of own subgroup and of intersection of subgroups. During the study of isomorphism and homomorphism of groups it is necessary to emphasize that isomorphic groups are completely coincident in terms of their algebraic properties and the relation of isomorphism is a relation of equivalence on the set of groups. Homomorphism of groups reveals only some similarity of some algebraic properties of the groups.

The results and their discussion. The supposed form of teaching the course of linear algebra needs an enough amount of study hours for mastering the discipline. Some years ago it was possible and the results of such educational process were essentially better, than now. Our experience tells us, that present-day first-year students are unable to master the substantial parts of discipline by their own.

Conclusions. The proposed detailed study of linear algebra and especially the beginnings of theory of groups leads the students to the facilitation of mastering such important concepts of algebra as concepts of a ring, a field, a vector space. It develops the abstract mathematical thinking of students. It helps to establish interdisciplinary connections between linear algebra and other mathematical and physical disciplines.

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THE PHENOMENON OF “READINESS” OF PROSPECTIVE PHYSICAL EDUCATION TEACHERS FOR MONITORING PHYSICAL CONDITION OF PRIMARY SCHOOL CHILDREN

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Abstract. *The article is devoted to the problem of training prospective teachers of physical education for monitoring the physical condition of primary school children. For determining the theoretical foundations of the research the methods of analysis, synthesis, comparison, systematization of scientific works on the problem under study were used. The author argues that one of the main aspects of physical education should*